Ultra-low-frequency electrodynamics of the magnetosphere-ionosphere interaction

A. V. Streltsov and W. Lotko

Thayer School of Engineering, Dartmouth College, Hanover, New Hampshire, USA

Received 28 August 2004; revised 10 May 2005; accepted 16 May 2005; published 13 August 2005.

[1] The results presented in this paper provide an explanation for electromagnetic oscillations with frequencies much less than the fundamental eigenfrequency of the magnetosphere measured in the regions where the ionospheric conductivity is low and a small-amplitude, large-scale electric field in the ionosphere exists. This study is based on numerical simulations of a reduced two-fluid MHD model describing propagation of dispersive Alfvén waves in the highly inhomogeneous magnetospheric plasma and interaction between these waves and the active ionosphere. Simulations show that electromagnetic oscillations with frequency below 10 mHz can be generated by a strongly nonlinear interaction between magnetic field-aligned currents and the ionosphere called ionospheric feedback instability. To produce oscillations in ULF frequency range, this mechanism requires neither very stretched geometry of the ambient magnetic field nor coupling between shear and slow MHD waves. These features of the ionospheric feedback mechanism makes it particularly suitable for the explanation of the ULF oscillations detected on midlatitude and low-latitude, nightside field lines when the background ionospheric conductivity is low and the magnitude of the perpendicular electric field in the ionosphere is small.


1. Introduction

[2] Electromagnetic oscillations with frequencies less than 10 mHz are frequently observed in the nightside, high-latitude auroral zone with ground-based magnetometers and radars [e.g., Samson et al., 1992b; Lotko et al., 1998; Ruohoniemi et al., 1991; Fenrich et al., 1995]. Sometimes the spectrum of these oscillations has a discrete character with distinct peaks at 0.8, 1.3, 1.9, and 3.5 mHz. The fact that at high latitudes these frequencies can match the fundamental eigenfrequency of shear Alfvén waves standing between the conjugate ionospheres along magnetic field lines stretched into the magnetotail [Rankin et al., 2000; Lui and Cheng, 2001] provides a rationale to interpret them as a manifestation of internal magnetospheric phenomena like shear Alfvén field line resonances (FLRs) and global magnetospheric cavity/waveguides waves [Samson et al., 1992a; Walker et al., 1992].

[3] Recent observations show that ULF oscillations occur more extensively than in the auroral zone. They have been detected at subauroral latitudes inside the so-called Subauroral Polarization Stream (SAPS) region [Foster and Burke, 2002] located around \( L = 3 - 4 \) magnetic shells. Also, they have also been detected at even lower latitudes, e.g., near the \( L = 1.6 \) magnetic shell [Francia and Villante, 1997], where their frequencies are far below the eigenfrequencies of the corresponding magnetic field lines. To explain this discrepancy, models of electromagnetic waves in the magnetosphere with additional degrees of freedom have been proposed, for example, a model that includes coupling between shear and slow MHD modes [Lu et al., 2003].

[4] The fact that the oscillations with the same ULF frequencies have been simultaneously detected in the solar wind plasma density, measured by the Wind spacecraft far upstream from the Earth, and in the oscillations of the magnetic field measured by the Geostationary Operational Environmental Satellite (GOES) 8 spacecraft in geosynchronous orbit inside the dayside magnetosphere [Kepko et al., 2002; Kepko and Spence, 2003] suggests that they may be not related to the eigenfrequencies of the magnetosphere but rather are directly driven by some external driver (for example, solar wind). This hypothesis was investigated by Streltsov and Foster [2004] who successfully modeled observations from the Millstone Hill radar in terms of surface Alfvén waves generated by an external driver on a steep transverse gradient in the background plasma density in the equatorial magnetosphere.

[5] In all these studies the ULF oscillations considered are interpreted as shear Alfvén waves generated as a consequence of the magnetosphere-ionosphere interaction, where the wave frequency is defined by the magnetosphere or some external driver located beyond the magnetosphere, for example, in the solar wind. In this paper we investigate the role of the ionosphere in the generation of Alfvén waves...
with unusually low frequencies. In particular, we numerically model the dynamics of dispersive Alfvén waves generated by the ionospheric feedback instability (IFI) introduced by Atkinson [1970].

[6] This instability may develop when there is a perpendicular electric field in the ionospheric E layer and the ionospheric conductivity is relatively low. Under these conditions a field-aligned current in the she Alfvén wave interacting with the ionosphere can locally enhance the conductivity by precipitating electrons in the $E$ layer. This increment in conductivity reduces Joule dissipation of the electric field in this location and releases free energy in the form of a field-aligned current propagating from the ionosphere. The contribution from this current can increase the magnitude of the reflected wave. If the wave is standing inside some resonant cavity in the magnetosphere, instability then develops. The energetic of the IFI was studied in detail by Lysak and Song [2002].

[7] There are two resonant cavities in the magnetosphere where the ionospheric feedback instability has been studied. One is formed by the entire closed magnetic flux tube bounded by the conjugate ionospheres, the so-called global field line resonator [Atkinson, 1970; Sato, 1978; Watanabe et al., 1993; Pokhotelov et al., 2002]. Another cavity, the so-called ionospheric Alfvén resonator (IAR), is formed by the ionosphere and the strong gradient in Alfvén speed in the low-altitude magnetosphere, where upward propagating Alfvén waves are partially reflected [Polyakov and Rapoport, 1981; Trakhtengertz and Feldstein, 1984, 1991, Lysak, 1991; Pokhotelov et al., 2000]. The main goal of this paper is to demonstrate that the frequencies of the electromagnetic oscillations generated by the IFI in the magnetosphere can be significantly lower than the eigenfrequencies of the corresponding FLRs, and they are mostly defined by the parameters of the ionosphere.

2. Model

[8] This study is based on a reduced two-fluid MHD model describing dynamics of dispersive Alfvén waves in the warm magnetospheric plasma [Chmyrev et al., 1988]. Warm plasma means that the plasma $\beta \ll 1$ but $\beta_e$ (electron $\beta$) can be smaller or larger than the electron to ion mass ratio, $m_e/m_i$. In such a plasma the effects of the finite electron and ion temperature are important in the equatorial magnetosphere [Hasegawa, 1976], and the effect of the finite electron mass is important near the ionosphere [Goertz and Boswell, 1979]. All of these effects are included in the physical model, which is described in detail together with a numerical algorithm used for its implementation in several recent papers [e.g., Streltsov et al., 1998; Streltsov and Lotko, 2003].

[9] The simulations are performed in the axisymmetric computational domain formed by a dipole flux tube, limited in latitude by the $L = 7.25$ and $L = 8.25$ magnetic shells and in altitude by the conducting ionospheres. The ionospheric boundaries are located at the altitude 120 km where the perpendicular electric field, $E_{\perp}$, is related to the parallel current density, $j_{\parallel}$, through Ohm’s law and current continuity [e.g., Lysak, 1990]:

$$\nabla \cdot (\Sigma P E_{\perp}) = \pm j_{\parallel}. \quad (1)$$

Here $\Sigma P$ is the height-integrated Pedersen conductivity and the plus/minus sign applies to the northern/southern ionosphere.

[10] The electrostatic boundary condition (1) neglects in the two-dimensional (2-D) model the inductive response of the ionosphere, which couples shear Alfvén and fast compressional waves via the rotational Hall current [Yoshikawa and Itonaga, 2000]. This coupling may be neglected when

$$\frac{n_0 \Sigma_P \lambda_{\perp}}{1 + \cosh(k_D d)} \left( 1 + \frac{\Sigma_P}{\Sigma_P^R} \right) \ll 1,$$

where $f = \omega / 2\pi$, $\lambda_{\perp} = 2\pi / k_{\perp}$, and $d = 120$ km is the height of the ionospheric $E$ layer. This inequality is well satisfied for the low frequency ($f \leq 10$ mHz), relatively small-scale ($\lambda_{\perp} \approx 10$ km) Alfvénic disturbances and relatively low ionospheric conductivity states ($\Sigma_P \leq 5$ mho) considered here.

[11] Ionospheric feedback occurs when $\Sigma_P = \Sigma_P(j_{\parallel})$ in (1). It is modeled by expressing $\Sigma_P$ via plasma density, $n_E$, averaged over the effective thickness of the $E$ layer. In this study $\Sigma_P = M_P n_E e / \cos \vartheta$, where $M_P = 10^4$ m$^2$/sV is the ion mobility, $h = 20$ km is the effective thickness of the $E$ layer [Miura and Sato, 1980]. $e$ is the elementary charge, and $\vartheta = 11^\circ$ is the angle between the normal to the ionosphere and the $L = 7.75$ dipole magnetic field line at the 120-km altitude. Dynamics of $n_E$ depends on the field-aligned current density via a convective term in the density continuity equation:

$$\frac{\partial n_E}{\partial t} = \frac{j_{\parallel}}{e h} + \alpha (n_{E0}^2 - n_E^2). \quad (2)$$

Here $\alpha = 3 \times 10^{-7}$ cm$^3$/s [Nygrén et al., 1992] is the coefficient of recombination and $n_{E0}$ is an equilibrium density in the $E$ layer.

[12] Some studies of the ionosphere feedback mechanism at high latitudes include additional source term representing effect of multiple ionization of the ionosphere by energetic electrons [Sato, 1978; Miura and Sato, 1980; Pokhotelov et al., 2002]. This effect is important, for example, when the dynamics of discrete auroral arcs associated with strong parallel potential drops is modeled. The focus of this study is ultra-low-frequency geomagnetic pulsations which are observed at high, middle, and low latitudes and associated with shear Alfvén waves curried by the cold plasma. Thus the effect of the additional ionization of the ionosphere by energetic auroral electrons is not included in our model.

[13] The geomagnetic field in the domain is defined as $B_0 = B_\star (1 + 3 \sin^2 \theta) 0.5/2 1.5 r^2$, where $B_\star = 31000$ nT, $\theta$ is a colatitudinal angle, and $r$ is a geocentric radial distance measured in $R_E = 6371.2$ km. The background plasma density is modeled as

$$n_0 = \begin{cases} a_1 (r - r_1) + a_2 & \text{if } r_1 < r < r_2 \\ b_1 e^{-20(r-r_1)} + b_2 r^{-4} + b_3 & \text{if } r > r_2 \end{cases} \quad (3)$$

Here $r(\mu)$ is a radial distance to the point on $L = 7.75$ magnetic field line with that particular $\mu$ value, $r_1 = 1 + 120/R_E$, $r_2 = 1 + 320/R_E$, and the constants $a_1$, $a_2$, $b_1$, $b_2$, and $b_3$ are chosen to provide some particular values of the plasma density at the altitude 120 km ($E$ layer maximum),
320 km (F layer maximum), and in the equatorial magnetosphere. In this paper we present results from simulations where the density in the $E$ layer has a magnitude of $3 \times 10^4$ cm$^{-3}$ (corresponding to $\Sigma_p = 1$ mho) and $9 \times 10^4$ cm$^{-3}$ (corresponding to $\Sigma_p = 3$ mho). Several values of the density in the equatorial magnetosphere will be considered: 0.25 cm$^{-3}$, 0.5 cm$^{-3}$, and 1.0 cm$^{-3}$.

[14] Dispersive Alfvén waves are initiated in the simulations by introducing small-scale disturbances of the density in the $E$ layer of the northern ionosphere where a large-scale perpendicular electric field exists. This “equilibrium” field is found by solving Laplace’s equation for the scalar potential, $\phi$, in the ionosphere:

$$\nabla \cdot (n_{E0} \nabla \phi) = 0,$$

with Dirichlet boundary conditions $\phi|_{L=7.25} = 0$, $\phi|_{L=8.25} = -4$ kV and $\phi|_{L=7.25} = 0$, $\phi|_{L=8.25} = -8$ kV. These boundary conditions provide maximum values of the background electric field, $E_{L=0} = -\nabla _L \phi$, of 25 mV/m and 50 mV/m in the ionosphere, respectively. Inside the computational domain $\phi$ is constant along the dipole magnetic field lines from the one ionosphere to another.

[15] Initial density disturbances inside the $E$ layer are modeled as $n = n_e \cos(2\pi (L - 7.75)) / (L - 7.75)^2$. The parameter $n_e$ defines amplitude of the disturbance which deviates from $n_{E0}$ by 10% or 50%. Simulations presented in the next section of this paper showed us that the amplitude of the initial density disturbance does not define the final (saturated) state of the M-I system but it is define how fast the system achieves the saturation. Thus in the most part of simulations demonstrated in the paper the model was initiated with a large-amplitude (50%) density disturbance.

[16] The parameter $L$ defines the perpendicular wave number and size of the initial wave packet across the ambient magnetic field. Two values of $L$ are considered in this paper: $L = 0.05$ ($L_2 - L_1$), which defines waves with the perpendicular wavelength of 8 km at the altitude 120 km, and $L = 0.1$ ($L_2 - L_1$), which defines waves with the perpendicular wavelength of 16 km at the altitude 120 km.

3. Results

[17] Theoretical and numerical studies of the ionospheric feedback mechanism by Trakhtengertz and Feldstein [1991], Lysak and Song [2002], and Streltsov and Lotko [2004] demonstrate that the instability develops when the background ionospheric conductivity is low. This condition is also confirmed by the simulations performed in this paper. Figure 1 presents two sets of snapshots of the parallel current density, $j_{L0}$, taken with an interval of 15.7 s from the simulations with $\Sigma_{p0} = 1$ mho (on the left) and $\Sigma_{p0} = 3$ mho (on the right). The small-scale Alfvén waves are initiated in both cases with 8-km density disturbances in the ionosphere, where the perpendicular electric field, $E_{L=0}$, has a magnitude of 25 mV/m. The ambient density in the magnetosphere, $n_m$, is set equal to 0.25 cm$^{-3}$, and all other background parameters in these simulations are the same.

[18] The fact that IFI develops only when the ionospheric conductivity is relatively low can be explained by the effect of the recombination. Indeed, for a some fixed set of the ionospheric parameters, like the effective thickness of the $E$ layer and the ion mobility, the low state of the ionospheric conductivity corresponds to the low plasma density, $n_{E0}$. Also, losses of the charged particles in the ionosphere due to the recombination in (2) are proportional to $n_{E0}$ $\Sigma_{p0}$ (here $\Sigma_{p0}$ is a disturbed part of the ionospheric plasma density and it is assumed that $\Sigma_{p0} \ll n_{E0}$). Thus the small magnitude of $n_{E0}$ means small losses due to the recombination. Obviously the same effect can be achieved by adjusting the magnitude of the recombination coefficient $\alpha$ in (2).

[19] To check this suggestion, a simulation was performed with the same background parameters as in the low-conductivity case ($n_{E0} = 3 \times 10^4$ cm$^{-3}$) illustrated in the left panels in Figure 1, except that the magnitude of the recombination coefficient was set equal to $9 \times 10^{-7}$ cm$^3$/s. This value is 3 times larger that the one used before so the magnitude of the loss term in (2) in this simulation is equal to the magnitude of this term in the high-conductivity case ($n_{E0} = 9 \times 10^4$ cm$^{-3}$) considered before. The simulation reveals that the instability does not develop in this case and the temporal dynamics of the MI system closely resembles dynamics of the system simulated in the high-conductivity case. Because the recombination inside the $E$ layer depends on the electron temperature and this parameter does not change in our model, we will use a fixed value of the recombination coefficient $\alpha = 3 \times 10^{-7}$ cm$^3$/s [Nygren et al., 1992] through the rest of the paper.

[20] This study is focused on the frequencies of oscillations generated in the coupled magnetosphere-ionosphere system by the ionospheric feedback mechanism. These frequencies will be determined by measuring the temporal variation of $j_{L0}$ at two fixed points on the $L = 7.75$ magnetic field line. The first point is located in the southern ionosphere at the bottom boundary of the computational domain. The second point is located at an altitude $\approx 1500$ km above the southern ionosphere, near the effective upper boundary of the ionospheric Alfvén resonator, as estimated for these background parameters by Streltsov and Lotko [2004].

[21] Figure 2 shows time variations of $j_{L0}$ in these two locations in the simulations with $E_{L=0} = 25$ mV/m, $n_m = 1$ cm$^{-3}$, and $\Sigma_{p0} = 1$ mho in the top panel in this figure and with $\Sigma_{p0} = 3$ mho in the bottom panel. The solid lines represent temporal dynamics of $j_{L0}$ in the southern ionosphere and the dashed lines represent dynamics of $j_{L0}$ at the altitude 1500 km above the southern ionosphere. The major result shown in this figure is that the frequency of oscillations produced by the ionospheric feedback instability is much lower than the frequency of the pure magnetospheric oscillations observed in the higher-conductivity case.

[22] The frequency of oscillations simulated in the high-conductivity case (lower panel in Figure 2) is 12.76 mHz. This frequency is very close (with a relative error <2%) to the fundamental eigenfrequency of 12.98 mHz of the Alfvén wave with this perpendicular wavelength standing along $L = 7.75$ magnetic field line, calculated from the eigenvalue equation derived by Streltsov [1999]. This equation includes dispersive effects of the finite temperature of the magnetospheric electrons and ions and the finite electron inertia. The dispersive effects are very important in these simulations because, for example, for the wave with the perpendicular wavelength of 8 km in the ionosphere the
Figure 1. Dynamics of the parallel current density in the simulation with $\Sigma_{P0} = 1$ mho (left) and with $\Sigma_{P0} = 3$ mho (right).
The ion Larmour radius correction to the parallel group velocity is \((1 + (k_i \rho_i)^2)^{1/2} = (1 + 0.916)^{1/2} = 1.384\) in the equatorial magnetosphere on the \(L = 7.75\) magnetic field line (for the magnetospheric parameters considered in this study). Also, for comparison, the fundamental eigenfrequency of the nondispersive Alfvén waves standing along the same magnetic field line is 9.74 mHz.

[23] The frequency of the oscillations obtained in the low conductivity case (top panel in Figure 2) is 4.79 mHz which is about 2.5 times less than the fundamental eigenfrequency of the \(L = 7.75\) magnetic field line. Also in this case, the simulation reveals higher frequency oscillations at the ionospheric altitude (solid lines). These are smaller-scale and higher-frequency Alfvén waves generated by the IFI inside the ionospheric Alfvén resonator (IAR) [Polyakov and Rapoport, 1981; Trakhtengertz and Feldstein, 1991; Lysak and Song, 2002]. The parallel current density in these waves has a maximum in the ionosphere and minimum near the upper boundary of the IAR [Streltsov and Lotko, 2004]; this is why they are barely evident at the altitude 1500 km (dashed lines).

[24] The goal of this study is to determine how the coupled magnetosphere-ionosphere system regulates the frequency of ULF electromagnetic oscillations when the system is feedback unstable (the ionospheric conductivity is low) and when it is stable (the ionospheric conductivity is high). We start the investigation with the case when the ionospheric conductivity is high.

### 3.1. High Ionospheric Conductivity

[25] The simulations show that in this case the oscillations are determined by the classical magnetospheric Alfvén resonator with frequency defined by the length of the magnetic field line, magnitude of the Alfvén speed along it (in particular, by its magnitude in the equatorial magnetosphere), perpendicular wavelength, and plasma temperature. The last two parameters are important for the eigenfrequency of the small-scale waves because these parameters define dispersion which significantly affects the parallel group velocity of the waves in the magnetosphere [Streltsov et al., 1998]. At the same time the eigenfrequency of the of the magnetospheric resonator does not depend on parameters of the ionosphere such as a perpendicular electric field in it.

[26] Figure 3 shows dynamics of \(j_{||}\) measured in the southern ionosphere on the \(L = 7.75\) magnetic field line in simulations with \(\Sigma p_0 = 3\) mho. The top panel shows results obtained in the simulations with the density in the equatorial magnetosphere \(n_m = 0.5 \text{ cm}^{-3}\) and the bottom panel shows results from simulations with \(n_m = 1.0 \text{ cm}^{-3}\). The dashed lines represents results from simulations where the background electric field, \(E_{\perp 0}\), has a magnitude of 25 mV/m in the ionosphere, and solid lines represents results from simulations where \(E_{\perp 0} = 50 \text{ mV/m}\) in the ionosphere. In all of these simulations Alfvén waves were initiated by the ionospheric density disturbance with a perpendicular wavelength of 8 km and an amplitude of 0.5 \(n_{E0}\).

Figure 2. (top) Parallel current density measured on \(L = 7.75\) magnetic field line in the southern ionosphere (solid lines) and at the altitude 1500 km above it (dashed lines) in the simulation with \(E_{\perp 0} = 25 \text{ mV/m}, n_m = 1 \text{ cm}^{-3}\), and \(\Sigma p_0 = 1\) mho. (bottom) Parallel current density measured at the same locations in the simulation with \(\Sigma p_0 = 3\) mho.

Figure 3. (top) Parallel current density measured on \(L = 7.75\) magnetic field line in the southern ionosphere in simulation with \(\Sigma p_0 = 3\) mho, \(n_m = 0.5 \text{ cm}^{-3}\), \(E_{\perp 0} = 25 \text{ mV/m}\) (dashed lines, scales on the left) and \(E_{\perp 0} = 50 \text{ mV/m}\) (solid lines, scales on the right). (bottom) Parallel current density measured at the same location in the simulation with \(n_m = 1.0 \text{ cm}^{-3}\).
The frequency of oscillations simulated when \( n_m = 0.5 \text{ cm}^{-3} \) (top panel in Figure 3) is 16.75 mHz for both \( E_{L0} = 25 \text{ mV/m} \) and \( E_{L0} = 50 \text{ mV/m} \). This value is close to 17.64 mHz, which is the fundamental eigenfrequency of the Alfvén wave with a perpendicular wavelength of 8 km in the ionosphere standing along the \( L = 7.75 \) magnetic field line. This eigenfrequency is calculated for the simulation parameters from the eigenvalue equation given by Streltsov [1999]. The same conclusion holds for the oscillations simulated with \( n_m = 1.0 \text{ cm}^{-3} \) (bottom panel in Figure 3). Their frequency is 12.76 mHz, which is close to the corresponding fundamental eigenfrequency of 12.98 mHz, and this frequency does not depend on the amplitude of \( E_{L0} \) in the ionosphere.

The dependence of the frequency of oscillation on the perpendicular wavelength of the initial density disturbance for the case of high ionospheric conductivity is illustrated in Figure 4. Here the solid line shows dynamics of \( j_B \) in the southern ionosphere on the \( L = 7.75 \) magnetic field line initiated by an ionospheric density disturbance with perpendicular wavelength 16 km; the dashed line illustrates the case when the perpendicular wavelength of the ionospheric density disturbance is 8 km. Other parameters used in these simulations are \( \Sigma_{P0} = 3 \text{ mho} \), \( n_m = 1 \text{ cm}^{-3} \), and \( E_{L0} = 25 \text{ mV/m} \) in the ionosphere. The frequency of the simulated 8-km oscillations is 12.76 mHz and the corresponding eigenfrequency of the magnetosphere is 12.98 mHz. The frequency of the simulated 16-km oscillations is 10.77 mHz and the corresponding eigenfrequency is 10.71 mHz.

The results illustrated in Figures 3 and 4 demonstrate good, quantitative agreement between frequencies of the coupled magnetosphere-ionosphere system simulated with a 2-D nonlinear numerical model and the fundamental eigenfrequencies of the magnetosphere calculated for the same background parameters from the 1-D linear eigenvalue equation. This finding confirms that (1) the 2-D nonlinear numerical algorithm used in this study adequately represents major features of the coupled MI system, and (2) for the case when the ionospheric conductivity is relatively high and IFI does not develop, the frequencies of ULF electromagnetic oscillations are defined in toto by the parameters of the magnetosphere.

### 3.2. Low Ionospheric Conductivity

[30] Now we consider the case when the Pedersen conductivity in the \( E \) region is low and the ionospheric feedback instability takes place. Simulations demonstrate that in this case, the frequency of oscillation does not depend on the magnetospheric parameters. Figure 5 illustrates the dynamics of \( j_B \) at the altitude 1500 km above the southern ionosphere on the \( L = 7.75 \) magnetic field line obtained in simulations with \( \Sigma_{P0} = 1 \text{ mho}, E_{L0} = 25 \text{ mV/m} \) in the ionosphere, and an initiate ionospheric density disturbance with perpendicular wavelength 8 km and amplitude 0.5 \( n_3 \). The solid line illustrates the case when \( n_m = 1.0 \text{ cm}^{-3} \) and the dashed line illustrates the case when \( n_m = 0.5 \text{ cm}^{-3} \). The frequencies of the oscillations in these two different simulations are essentially identical (4.79 mHz).

[31] Figure 5 also shows that the amplitude of oscillations generated by the ionospheric feedback mechanism saturates at some level. To understand the saturation mechanism, let us consider dynamics of \( j_B \) in the southern ionosphere in the simulation with \( \Sigma_{P0} = 1 \text{ mho}, E_{L0} = 25, n_m = 1.0 \text{ cm}^{-3} \) initiated by 8-km ionospheric density disturbances with an amplitude of 0.1 \( n_3 \). This dynamics is shown in Figure 6 where the one-dimensional plot shows \( \Sigma_P \) and \( E_L \) in the southern ionosphere at time 1000 s (along the horizontal line in the 2-D plot). This plot demonstrates that the saturation occurs when the system is in a strongly nonlinear state: the magnitudes of the disturbed \( E_L \) and \( n_E \) are comparable to their background values.

[32] It should be mentioned here that strong disturbances of the density inside the \( E \) layer can also affect density in the low magnetosphere. Thus for example, radar observations in the auroral zone reported by Doe et al. [1993] demonstrate existence of the extended along the magnetic field lines density cavities associated with the downward current channels. Such modification of the density in the magnetosphere can affect structure and impedance of the Alfvén wave line.
waves interacting with the ionosphere. Because shear \( \text{Alfvén waves do not produce significant density disturbance, this effect can be taken into account by including in the model slow MHD waves, as was done by Lu et al. [2003]. These waves are not incorporated in the current model, first, because coupling between shear and slow modes requires relatively large amplitude of the fields and currents and second, because of some radar observations of the ionospheric density at high latitudes [e.g., Shepherd et al., 1998] show large-amplitude disturbances of the density completely localized inside the ionospheric \( E \) region, as it is described by the present model.}

\[ [35] \text{The amplitude of the disturbed part of the density obviously cannot be larger than the background value; otherwise, the total density in the ionosphere would be negative. To prevent this nonphysical effect, the algorithm has a threshold for the minimal value of the ionospheric density. Computations show very little difference in the amplitudes and dynamics of the solutions when this threshold is set equal to 100 cm\(^{-3}\) (\( \Sigma_p = 3.33 \times 10^{-3} \text{ mho} \)) and 10 cm\(^{-3}\) (\( \Sigma_p = 3.33 \times 10^{-4} \text{ mho} \)). Other mechanisms that may contribute to the saturation include recombination, parallel collisional resistivity of the plasma, and numerical dissipation related to the finite difference approximation of the model equations.}

\[ [34] \text{To our knowledge, all previous theoretical investigations of the IFI [e.g., Sato, 1978; Trakhentertz and Feldstein, 1984, 1991; Lysak, 1991; Pokhotelov et al., 2000; Lysak and Song, 2002] have been performed assuming that the magnitude of the disturbed part of the density is much smaller than its background value and the strongly nonlinear dynamics of the IFI instability evident in Figure 6 and other simulations presented in this paper has not been previously studied. The dispersion relation describing IFI in linear 2-D case (without Hall conductivity and without effect of the additional ionization of the ionosphere by energetic electrons) has the form [Miura and Sato, 1980]}

\[ \omega = \frac{k_z M_p E_{z0}}{1 + Z \Sigma_p} - i \omega n \omega E_0, \]  

where \( Z \) is a magnetospheric impedance connecting \( j \parallel \) and \( E_\perp \) in the relation \( \Sigma_{\parallel} = \nabla \cdot E_\perp \). In general \( Z \) is a complex function depending on \( \omega \). One form of \( Z \) inside the magnetospheric resonator is given by Lysak and Song [2002] as

\[ Z = \frac{1}{\Sigma_{\perp}} \frac{1 + R e^{i \omega T}}{1 - R e^{i \omega T}}, \]  

Here \( \Sigma_{\perp} = 1/\mu_0 v_A \) is the Alfvén conductance, \( R \) is a reflection coefficient for the wave perpendicular electric field, and \( T \) is the travel time to the reflection point. Thus when the disturbances of the ionospheric density is much smaller than its background magnitude it is possible to estimate \( T \) and \( R \) (for example, by using expression \( R = (\Sigma_{\perp} - \Sigma_p)/(\Sigma_{\perp} + \Sigma_p) \) given by Mallinckrodt and Carlson [1978]) and to find \( \omega \) by solving equation (5) numerically. For example, in a limiting case when \( \Sigma_{\perp} \approx \Sigma \) near the ionosphere the reflection coefficient becomes close to 0 and the frequency of the wave generated by the ionospheric feedback mechanism and propagating to the magnetosphere is \( \omega = k_z M_p E_{z0} 2.0 - i \omega n \omega E_0 \). (The instability does not develop in this case (without reflection) because the imaginary part of \( \omega \) is negative.)

\[ [35] \text{Unfortunately, an amplitude of the density disturbances shown in Figure 6 makes it hard to use this linear theory for the explanation of the frequency of the saturated oscillations. A rigorous theoretical derivation of the nonlinear dispersion relation for the IFI that self-consistently describes dynamics of \( \Sigma_{\perp} j_\parallel \) and \( E_\perp \) is a task beyond the scope of this paper. Here our goal is to investigate by means of numerical simulations the dependence of coupled MI dynamics on two ionospheric parameters: the magnitude of the background convective field \( E_{z0} \) and the perpendicular wavelength of the initial disturbance.}
[16] Figure 7 shows dynamics of $j_\parallel$ at the altitude 1500 km above the southern ionosphere on the $L = 7.75$ magnetic field line obtained in simulations with $\Sigma_{P0} = 1$ mho and $n_m = 1.0$ cm$^{-3}$. The dashed line illustrates the case when $E_{1.0} = 25$ mV/m in the ionosphere and the solid line illustrates the case when $E_{1.0} = 50$ mV/m. The IFI is initiated by an ionospheric density disturbance with perpendicular wavelength 8 km and amplitude of $0.5 n_{E0}$. The simulated frequency of oscillation for the case with $E_{1.0} = 25$ mV/m is 4.79 mHz; the frequency of oscillation for the case with $E_{1.0} = 50$ mV/m is 5.58 mHz.

[17] Figure 8 shows the dynamics of $j_\parallel$ at the altitude 1500 km above the southern ionosphere on the $L = 7.75$ magnetic field line obtained in simulations with $\Sigma_{P0} = 1$ mho, $n_m = 1.0$ cm$^{-3}$, and $E_{1.0} = 50$ mV/m in the ionosphere. The dashed line illustrates the case when the IFI is initiated with the ionospheric density disturbance with perpendicular wavelength 8 km and the solid line illustrates the case when it is initiated with a 16-km ionospheric disturbances. The frequency of oscillations initiated with 8-km waves is 5.58 mHz, and the frequency of oscillations initiated with 16-km waves is 4.79 mHz. The last value of frequency is the same for the case with $E_{1.0} = 25$ mV/m and an initial perpendicular wavelength of 8 km.

[18] As the last point of interest, we have checked the dependency of the development of the instability on the magnitude of the recombination coefficient $\alpha$. Simulations with $\Sigma_{P0} = 1$ mho, $n_m = 1.0$ cm$^{-3}$, and $E_{1.0} = 50$ mV/m in the ionosphere initiated with the ionospheric density disturbance with perpendicular wavelength 8 km do not show any variation in frequency of oscillations when $\alpha$ is varying in the range from $2 \times 10^{-7}$ to $6 \times 10^{-7}$ cm$^3$/s. Although the same simulations reveal that the saturation state of the instability occurred much later in the simulation with higher $\alpha$ compare with the simulation with smaller $\alpha$. These simulations together with the simulations discussed in the beginning of this section let us to conclude that the recombination defines the threshold and the growth rate of the instability but do not affect a period of the saturated oscillations.

4. Conclusions

[19] This paper presents results from a numerical study of strongly nonlinear electrodynamics of ULF oscillations of the high-latitude, magnetosphere-ionosphere interaction. The study is based on a reduced two-fluid MHD model describing propagation of dispersive Alfvén waves in the highly inhomogeneous magnetospheric plasma and the interaction of these waves with an active $E$ region ionosphere. The main goal of this study is to identify the conditions under which this MI interaction supports electromagnetic oscillations with frequencies that is much lower than the fundamental eigenfrequency of the flux tube.

Figure 7. Parallel current density measured on $L = 7.75$ magnetic field line at the altitude 1500 km above the southern ionosphere in the simulation with $\Sigma_{P0} = 1$ mho, $E_{1.0} = 50$ mV/m (solid line) and $E_{1.0} = 25$ mV/m (dashed line).

Figure 8. Parallel current density measured on $L = 7.75$ magnetic field line at the altitude 1500 km above the southern ionosphere in the simulation with $\Sigma_{P0} = 1$ mho and $E_{1.0} = 50$ mV/m initiated by the ionospheric density disturbances with 8 km perpendicular wavelength (solid line) and 16 km perpendicular wavelength (dashed line).
[41] Results from the simulations confirm that when the background plasma density inside the ionospheric $E$ region is relatively high, implying a relatively high Pedersen conductivity, the MI system is stable with respect to the ionospheric feedback instability. In this case, the system response to an $E$ region perturbation of the equilibrium density is found to be a temporally decaying train of fundamental-mode field line oscillations. As described in many previous studies, the eigenfrequency of these oscillations is determined by the length of the magnetic field line and the distribution of the background Alfvén velocity along it; for latitudinally narrow oscillations, Alfvén wave dispersive effects associated with the finite plasma temperature and electron inertia also moderate the eigenfrequency. As expected for a field line resonance, the frequency of oscillation is determined primarily by the two-fluid dynamics of Alfvén wave in the magnetosphere and, for the parameters of our numerical studies, does not depend on the amplitude of the perpendicular electric field in the ionosphere or other ionospheric parameters. (We are referring here only to the behavior of relatively low-frequency Alfvén waves standing along field lines between the northern and southern ionosphere.) The high-frequency, resonant Alfvén waves (0.1–1 Hz) trapped in the low-altitude ionospheric Alfvén resonator are sensitive to ionospheric conditions as discussed by Trakhtengertz and Feldstein [1991], Lysak and Song [2002], and Streltsov and Lotko [2004].

[42] It is well known that when a slowly varying electric field permeates regions of relatively low ionospheric conductivity, $E$ region density perturbations can be amplified by the ionospheric feedback instability, leading to a formation of intense ULF electromagnetic disturbances. In case of relatively short perpendicular wavelengths and relatively large electric fields in the ionosphere, the characteristic frequency of nonlinear feedback-unstable oscillations defined by the ionosphere can become equal to the field line eigenfrequency. This is a resonance regime which has been investigated in a number of theoretical and numerical studies.

[43] In this study we find that the ionospheric feedback instability can successfully develop even with a much lower characteristic frequency than the fundamental eigenfrequency of the flux tube, particularly when the system dynamics is strongly nonlinear. The resulting oscillation frequency is constrained by the timescale of the interaction between field-aligned currents flowing into and out of the ionosphere and the ionospheric Pedersen currents that closes the field-aligned currents. So the frequency is regulated by the cross-field collisional dynamics of ions carrying the Pedersen current and, unlike a shear Alfvén field line resonance, it does not depend significantly on the density or temperature distribution in the magnetosphere. Not surprisingly, the frequency of oscillation is found to be proportional to the amplitude of the background electric field in the ionosphere and inversely proportional to the perpendicular wavelength or the horizontal width of the closure path for Pedersen currents.

[44] The results presented in this paper provide an appealing explanation for a class of meridionally localized, ULF oscillations with perpendicular scale sizes $\leq 10$ km at the altitude 100 km (or on the ground) and with frequencies much less than the fundamental eigenfrequency of the corresponding field lines (typically below 10 mHz). The interpretation proposed here is that the observed oscillations are not field line resonances but rather nonlinear feedback-unstable oscillations with frequency determined by the timescale for Pedersen current closure of the time-dependent field-aligned currents. This mechanism requires neither stretched magnetic geometry nor coupling between shear Alfvén and slow MHD waves to achieve what would otherwise be interpreted as an abnormally low eigenfrequency. These features of the proposed mechanism make it particularly suitable for explaining ULF oscillations detected in the midlatitude and low-latitude, nighttime ionosphere and magnetosphere, where the background ionospheric conductivity and the magnitude of the perpendicular ionospheric electric field are often small.

[45] Acknowledgments. The research was supported by NASA grant NNG04 GE22G, by NSF ATM-0095187 grant, and by the Sun-Earth Connection Theory Program grant NAG5-11735.

Arthur Richmond thanks Robert Lysak and two other reviewers for their assistance in evaluating this paper.

References


W. Lotko and A. V. Streltsov, Thayer School of Engineering, Dartmouth College, 8000 Cummings Hall, Hanover, NH 03755-9000, USA. (william.lotko@dartmouth.edu; streltsov@dartmouth.edu)